PROBLEMS AND SOLUTIONS.

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PROBLEMS FOR SOLUTION.

[N. B. Problems containing results believed to be new, or extensions of old results, are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known text-books, or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist the members of the Association with their difficulties in the solution of such problems.]

2949 (Corrected; see 1922, 29). Proposed by J. B. REYNOLDS, Lehigh University.

Find the lateral area of the cone with vertex at $(0, 0, h)$ and whose base is the epicycloid,
$$2x = a(3\cos\theta - \cos 3\theta), \quad 2y = a(3\sin\theta - \sin 3\theta).$$

2992. Proposed by AUGUSTUS BOGARD, Teresian University, Winona, Minn.

A semi-circle rotates at a uniform velocity about its diameter and slides along the line of that diameter at such a uniform rate as just to pass the full length of the diameter while making one revolution about it. Find the equation of the surface thus generated.

2993. Proposed by H. C. BRADLEY, Massachusetts Institute of Technology.

Let ABC be any triangle, and O the center of its circum-circle. Bisect the arcs AB, BC, and CA at F, D, and E. With F, D, and E as centers draw arcs passing in each instance through the adjacent corners of the triangle. Prove that these arcs intersect at the in-center of the triangle ABC.

2994. Proposed by R. M. MATHEWS, Wesleyan University.

Can the following construction be made without the use of a regulus? Construct a line which meets four given skew lines.

2995. Proposed by S. A. COREY, Des Moines, Iowa.

Give a geometric proof of each of the identities:
(*a*) $\cos(a + 2mx) = \cos a - 2\sin x[\sin(a + x) + \sin(a + 3x) + \cdots + \sin(a + (2m - 1)x)]$,
and
(*b*) $\sin(a + 2mx) = \sin a + 2\sin x[\cos(a + x) + \cos(a + 3x) + \cdots + \cos(a + (2m - 1)x)]$,
where m is a positive integer.

2996. Proposed by E. J. OGLESBY, Flushing, N. Y.

Given $u_1 = .2500$, $u_2 = .4113$, $u_3 = .4785$, $u_4 = .4965$, find x when $u_x = .4311$.

2997. Proposed by M. ZAMETKIN, Jamaica, N. Y.

Given $a = \sin 5°$, $b = \sin 49°$, and $c = \sin 87°$, prove that
$$\sin 73° = \frac{a^2 - b^2 + ac}{4a(a^2 - b^2 + ac) - (a - b + c)}.$$

2998. Proposed by F. M. GARNETT, Augusta, Ga.

A cube has removed from it a right pyramid whose base is a face of the cube and whose altitude is the altitude of the cube. How far from the base of the cube must a plane be passed parallel to the removed face so as to divide the remaining volume of the cube into two equal parts?